1. Preliminary words

This paper presents a dialogue between its two authors, **ET** (Enric Trillas), and **AS** (Alejandro Sobrino), concerning both the man and the engineer, scientist and thinker Lotfi Aliasker Zadeh (Baku, February 1921- Berkeley, September 2017), whom they both had the fortune of knowing personally.

**ET.** I learned about and read Zadeh’s 1965 paper Fuzzy Sets [11] in 1975, and I met him personally in July 1977, as of when we maintained a long, sincere friendship that lasted up to his passing away; the last emails we exchanged were at the end of June, 2017. I find impossible to evaluate the full value that those forty years of friendship have for me; I did learn a lot from the lectures I heard from Zadeh, and the papers of his that I read but, mainly, from the discussions and the conversations we used to have on wide variety of topics. For me, Lotfi A. Zadeh was an unforgettable master who, along with Karl Menger (Vienna 1902-Highland Park 1985), I believe are those who had the greatest influence on my research activities. Actually, the imprints of their works are clearly evident behind my own thought, for instance, Menger’s addendum to Occam’s Razor, or Zadeh’s view on the constraint imposed by trying to attain too much precision. For me Zadeh was up there among the most creative researchers of the 20th Century.

**AS.** I started reading Zadeh in 1985 and I met him in person for the first time in 1996, at the XXVI IEEE Multiple-Valued Logic Symposium that we organized at the University of Santiago de Compostela, Spain. My initial approach to the fuzzy logic was through the book, *Introducción a la lógica formal* (Introduction to Formal Logic) by the Spanish philosopher of logic, Alfredo Deaño, who, in a few lines, approached the principal features of what was, back then, a new type of logic. In a footnote he said that fuzzy logic attempts to explain imprecision in a precise way. Using a metaphor, he said that it is like an oral theory of stuttering offering explanations about stuttering but without stuttering. This phrase captivated me in such a way that I found myself desperate to learn more about the topic. As there was no such thing as the Internet, I audaciously wrote to Zadeh asking him for papers, which he kindly sent to me a few months later. His papers caused a double impression on me: one hand, they were so exquisitely written. If language is a window to our mind, I conjectured that Zadeh’s mind would be like his writing: exciting, creative, brilliant. On the other hand, his papers frequently illustrated mathematical or logical notions with examples using natural language. Although the topics dealt with were sometimes cryptic for a graduate in Philosophy, the examples provided the reader with accurate insights. Giving and selecting suitable examples is a sign of clear and distinctive intuition, which Zadeh undoubtedly possessed.
ET & AS. Hence this short paper can be neither a typical obituary, nor can it merely contain impartial and independent views. It endeavors to present a passionate picture of ‘the Lotfi A. Zadeh’ we knew, a number of hints on how, and in line with the previous comments, we see the work of one of the most creative researchers we have had the great opportunity of knowing personally, admiring him, and intellectually benefiting from both his thought and his work. Zadeh was not only an engineer and a scientist, but also an innovative thinker [3], and this paper is no more than a personal, respectful and warm tribute to his memory offered by two researchers, not only working into different fields, but belonging to different generations.

2. Some personal memories of Zadeh

ET. Zadeh had a great personal attractiveness; people approached him in a truly incredible way, one example of which is, perhaps, an extreme case which occurs in the spring of 1981 in Berkeley. After working all morning at Evans Hall, Zadeh and those of us that were there with him at the time, went for lunch to the ‘famous’ ‘Three Cs’ cafeteria, located outside the campus in Hearst Street. As the weather that day was so good, we sat at a table outside the cafeteria to eat, as Lotfi usually did back then, a frugal ‘crêpe’ and a cup of Darjeeling Tea. We were in the middle of a pleasant conversation when a beautiful, elegantly dressed lady came out from the cafeteria, walking in the direction of our table; we all gawked at her. Just as she was about to walk past us, Zadeh said to her, ‘Please, come and join us!’ And she did. After a talking for a while, and even if the conversation seemed to be between old friends, it became patently obvious that she and Zadeh had never met each other before.

Zadeh was a man who loved culture and beauty, as most of the artistic pictures he took over many years show, with photos not only of scientists and engineers, but also of dancers, writers, singers, politicians, etc. One day, in 1980, I was invited to lunch at Fay and Lotfi’s home in Mendocino Avenue. At that time the living room in the house was full of very large loudspeakers and, while waiting for Fay, Zadeh and I were talking about Italian Opera and its singers, something about which I was, and still I am, very passionate. Zadeh proceeded to put a record on and said to me, ‘This is an exam for you’, and asked me ‘What is the song and who is the singer? Then came my answer, ‘It’s Nessun dorma from Puccini’s Turandot, and the singer is Luciano Pavarotti’, and he said to me in a soft voice ‘We’re going to be good friends’. In fact, and for forty years, not only were we good friends, but loyal ones, too.

In private situations, Zadeh had a great sense of humor. Maybe it was in 1983 when Sergei Ovchinnikov, Elie Sanchez and I were invited to lunch at Fay and Lotfi’s home. Sergei had left the Soviet Union a few years before, and had only very recently discovered his books which had finally been sent to him from Moscow, after the inordinately long time the authorities took to scrutinize them, one by one. Lotfi presented us with three red shirts, identical to the one he was wearing, and asked us to put them on in order to take a group photo. But as Sergei attempted to gently refuse, Zadeh said ‘Sergei, please, put the shirt on, even if it is too radical for you!’ Sergei did as he was told, and the picture of the four turned out very nice; published in a paper by Sergei as well as in a book in homage to Zadeh [3]. It is just a shame you can’t make out the red in the black and white photograph.

Not only was Zadeh very kind with everyone, and, in particular with young researchers with whom he always engaged in conversation and advised in kind, simple terms, but he always gave his oral presentations without any attempt to show one iota of personal superiority over the audience. Only once did I ever see him lose his temper in public; it was at a seminar in Berkeley, back in the mid-eighties, when a young researcher presented his ideas in an excessively shocking and bombastic style. He quoted Henri Poincaré, but wrote in the blackboard ‘Pointcarré’. Zadeh suddenly leapt up from his seat towards the blackboard and, while rubbing out ‘Pointcarré’ and writing
'Poincaré', shouted earnestly at the speaker, ‘Be careful with spellings!’ The speaker blushed, finished off quickly and was never seen or heard from by us again.

In our conversations I did not always fully agree with Zadeh, but I always came out enriched by the discussion; he induced me to keep turning things over and over in my mind. One example of such an occasion is the following story.

Given a predicate \( P \) and its membership function \( m_P \), in order to represent the membership function of its antonym \( aP \), Zadeh maintained that this should be done by taking \( m_{aP}(x) = m_P(a + b - x) \); that is, by considering the symmetry \( s(x) = a + b - x \) in \([a, b]\), and defining \( m_{aP} = m_P \circ s \). Agreeing on the use of a symmetry to reflect the opposition between \( P \) and \( aP \), I tried to convince him that the symmetry should depend on \( P \); that it cannot be the same for all \( P \) in \([a, b]\); he refused to accept it, and all the examples he certainly managed to give me some good antonyms. It took me a while to realize that if the graph of \( m_P \) is either decreasing or non-decreasing, as was the case in all the examples given by Zadeh, just the symmetry \( a + b - x \) is fine, but that the problem arises when this graphic is not monotonic \(^9\). Knowing full well that in order to convince Zadeh I would have to provide him with a concrete counterexample, I decided to postpone the discussion. The next day and finally, the predicate ‘close to four’ in \([0, 10]\) whose antonym should be ‘far from four’, gave me the solution, since with the symmetry \( 10 - x \) we obtain a membership function corresponding to the predicate ‘close to six’. I immediately returned to broach the subject with Zadeh and, after joking with me “Well, but ‘close to six’ already implies ‘far from four’”, he accepted that the symmetry should be taken accordingly with the characteristics of \( P \), that it should be \( s_P \) but not only \( s \).

After what was certainly a pyrrhic victory, I duly forgot the question which, nevertheless, remained hidden in my subconscious up to when, many later, I returned to it through writing a paper, along with a number of colleagues, on computation with antonyms \(^7\). It was after I had computed the antonym of ‘close to four’ by using a symmetry in \([0, 10]\) decomposed into several parts, each one corresponding to the subintervals in which \( m_{a\text{round }4} \) is monotonic, constructing it like \( a + b - x \) but with \( a \) and \( b \) being the extremes of the corresponding subinterval, and obtaining a function easily recognizable as a membership function representing ‘far from four’. For me and soon or later, discussions with Zadeh always turned out to be fruitful.

Throughout Zadeh’s life, the efforts he made in continuously encouraging people to work on fuzzy logic were remarkable. Even if he almost always published papers with just his own signature (he liked to work alone), he never refused to discuss his results with anybody; his lectures were typically rounded off with, *Please, don’t hesitate to contradict me!* A highly frugal person, he never ate too much, smoked, or drank alcohol; his health was punished by the myriad trips around the world to deliver addresses at a multitude of conferences in different countries, and for many years; for instance, after the serious heart attack he suffered at the end of 2009, he told me that he had flown over 100,000 miles that year.

Zadeh enjoyed contact with people for different cultures and maintained a healthy respect for them all. He was not a nationalistic person; rather he preferred multi-cultural growth for people. The academic and industrial culture of fuzzy logic spread all over the world.

**AS.** Zadeh was an engineer who loved mathematics. He believed that mathematics provided an accurate analysis of both natural and anthropocentric problems. In fact, and as he explained, before 1964 he asked the prominent mathematicians, Herbert Robbins and Richard Bellman \(^4\), to develop a mathematical theory of sets with unsharp boundaries, a request which, owing to the numerous commitments that two mathematicians had back then, was not met. In 1964, Zadeh on his own, came up with the notion of fuzzy set specified though a membership function, which he published in 1965 \(^11\), and is currently one of the most cited papers from the 20\(^{th}\) Century.
Zadeh’s main concern of was with precisiation and formalization. Precisiation is to change vagueness, a linguistic-philosophical concept, into fuzziness, a measurable or scientific one.\[^5\] Formalization is to provide mathematical models to understand problems from science or humanities; as they usually show borderline cases, Zadeh advanced fuzzy logic, a logic that represents classes with unsharp edges.

Zadeh was a huge fan of mathematics and somewhat skeptical about the humanities. He demonstrated a certain degree of disdain with philosophical works emphasizing the vagueness of natural language, such as Wittgenstein’s *Philosophical Investigations*. He was also critical with Eleanor Rosh or George Lakoff’s approaches from cognitive science, as they did not use mathematical equations, but vague notions such as ‘category’ or ‘hedge’.\[^4\] He did not fully agree with approaches from Philosophy, as it addresses vagueness without introducing precision. In his view, and resorting to the previous metaphor, for him, Philosophy seemed to be a kind of stuttering theory that explains its subject while stuttering.

In any case, he contributed with a paper\[^12\] to the Philosophy Journal *Synthese*, in a special volume devoted to ‘On the logic semantic of vagueness’. In the ‘references’ section he quoted works from M. Black, W. V. Quine, K. Fine and C.G. Hempel, but in an unspecific way, without correspondence to specific entries in the text. It seems as if the philosophers inspired him with ideas, but not to realize them. In turn, Zadeh was not warmly received in the arena of Philosophy, where vagueness prevails over fuzziness and subjectivity overcomes objectivity. Philosophy pursues complexity and, by analyzing a problem, philosophers expect to uncover certain other problems, instead of solving the one that they are attempting to address; an attitude which is the complete antithesis of that of an engineer.

As an engineer, Zadeh was a pragmatic man seeking useful models to manage problems that admit solutions by precisiating the meaning of the words used to enunciate them. He provided inference rules for gathering plausible conclusions based on vague premises. Although the term ‘fuzzy logic’ is not always welcome in Mathematics or Philosophy, it has become a widely used paradigm and accepted in the fields of cognitive science and artificial intelligence. Qualified by some philosophers as a kind of techno-logic (a logic ‘just’ for technology) rather than a type of logic, it has been worthy of that denomination with diverse, bright and useful applications.

### 3. On what will remain associated with Zadeh’s name

**AS.** Surely, the best homage to the intellectual legacy of Lotfi A. Zadeh would be a critical approach to what he published; but probably his passing away is still too close in time to attempt to embark on such an intellectual task wherein, in any case, we should take into account the moment in which he introduced the new ideas. For instance, if the concept of *fuzzy set* was received as a genuine ‘intellectual shock’ in 1965, that of *Soft Computing* in the early nineties, beyond the relevance thereof, was something like *blowin’ in the wind*, in the words of Bob Dylan; at that time, hybrid methodologies were already starting to be seen as necessary tools for trying to reach some suboptimal solutions to computational problems that could not be addressed by any of them separately. Concerning *Computing with Words and Perceptions*\[^13\], not only did he return to and expand on the original goals of Fuzzy Logic, but he touched on one of the main shortcomings of Artificial Intelligence.

**ET.** Relating Zadeh’s name with its new, more central ideas depends on who is trying to do so; it is the *Quidquit recipitur ad modus recipientis recipitur* of Thomas of Aquinas. People grasp new concepts as their personal formation and interests allow them to do so, both not always completely coinciding with those of the introducers of these concepts; once a liquid is in a bottle, it adopts the bottle’s form. Anyone who is captivated by the ideas of Zadeh will grasp them in the way that they are able to do so, and in relation to the ongoing issues. Also, and at least at the very beginning, there were a good many important scientists in the USA who simply rejected these
ideas, going as far as demonstrating their loathing for them; a situation that changes, diminishing over time, owing to the technological successes attained by some of them and, mainly, those concerning Fuzzy Control reached after the pioneering work of Abe Mamdani (1942-2010) in the UK [8], and the industrial success in home appliances and more relevant technological achievements made by some important Japanese industries. Notwithstanding, and from very early, in Japan and Europe, and even in the old USSR, Zadeh’s new ideas were received with notable interest, and without the intellectual preemptions voiced against them by certain American scientists. In this regard, and far from the typical struggles between groups of scientists for capturing research funds from official agencies, it should not to be forgotten that in the mid-seventies the ‘Space Race’ was at full throttle, confronting the USA and the old USSR, and for which precision and precise computations were very important; the quest for not only studying imprecision but for benefiting from it, collided with the prevailing scientific interest in crisp methods.

AS. In the particular case of Spain, for instance, these ideas were first and more or less simultaneously but independently grasped in the areas of philosophy, mathematics and statistics; the first as a consequence of the interest in linguistic vagueness; the second through the possibility of extending the mathematical representation of the imprecise linguistic statements that Boolean algebra, classical set theory, does not permit; and the third through the non-realistic precision imposed on questions and answers in opinion polls. All of them were, actually, related with certain aspects of plain language.

ET. Fifty-three years after the 1965 paper Fuzzy Sets, there are six concepts that undoubtedly deserve to be linked with Zadeh’s name [14]. The first of these is the concept of Fuzzy Set, as a nebulous entity giving extension, in a given universe of discourse, to an imprecise predicate, and contextually represented by a specifically designed membership function. The second is the Algebra of Fuzzy Sets reached through their membership functions. The third is the concept of a Linguistic Variable. The fourth is the Compositional Rule of Inference. The fifth is the Principle of Extension, and the sixth is the idea of a Generalized Constraint. Without them it is difficult to imagine how fuzzy sets could have being applied to technology in a way as successful as it was and still is; and even less so, how Zadeh could have come up with the ideas on Computing with Words and Perceptions [13] (CwW). Let’s say something on each one of these six basic concepts.

The first and the second constituted the starting point for everything that followed in applying fuzzy set theory to both theoretical and practical problems, and also meant a departure from earlier attempts at the very idea of a fuzzy set. For instance, it was Karl Menger who introduced the concept of a ‘hazy set’ as a cloud of points with positive probability of belonging to a classical set, but Zadeh expanded on the notion (of which he was unaware in 1965) by liberating it from probability, since not all cases of uncertainty appearing in plain language correspond to a probabilistically measurable event; not all linguistic uncertainties are random and, hence, the supposition of the additive law is, at least, problematic. Additionally, Max Black (Baku 1909-Ithaca 1988) introduced the compatibility, or profile function, to represent an imprecise word, but neither Menger nor Black ever introduced an algebra with the new elements, as Zadeh did in his first 1965 paper; these algebras, further extended with more representations of the involved connectives than those initially presented by Zadeh, opened up the possibility of fuzzy sets being ‘operative’ in many different situations and depending on the contextual meaning of the words involved.

In fact, Zadeh accomplished what John von Neumann (Budapest 1903-Washington D.C 1957) called for back in the fifties [10], and that entailed considering the semantics of concepts, and not only syntactical formulas of a presumed universal validity. In addition, Zadeh introduced the armamentarium of Mathematical Analysis into these problems, as had also been called for by von Neumann. It should not be forgotten that Zadeh came from the time of Cybernetics, where Mathematical Analysis played an important role.
The sixth, even formulated by Zadeh many years after 1965, once again clearly entailed entering into the domain of representing plain language, where ‘x is P’ is “true”, does not have the same meaning as it is “probable”, or as it is “possible”, etc. These constrained statements are submitted to be computed by means of both the membership function $m$ and the measures for true, probable, possible, etc. In short, Zadeh took a step further along the pathway embarked on by George Boole in the mid-19th Century to represent linguistic statements, surpassing earlier attempts such as those of Menger and Black.

AS. Returning to the second for a while, this would also mean the penetration of fuzzy sets into plain language, where even the most simple connectives ‘and’, ‘or’, ‘not’, are not universal, as is the case with precise words (intersection, union and complement of sets); rather they vary contextually. In a long statement with several occurrences of such connectives, it cannot be assumed that they can all be represented by the same function. In fact, in his 1965 paper, *Fuzzy Sets* [11], Zadeh introduced several possibilities for representing these connectives, such as, for instance, and for the case of the conjunction, the functions minimum, product, and the Lukasiewicz operation, which were subsequently generalized to open up further possibilities for representing imprecise statements.

ET. With regard to the third, along with the sixth, these help greatly in dealing with common sense reasoning expressed in plain language, since the concepts involved therein, such as *Age*, are often not handled by means of numerical values but by ‘linguistic values’, such as *young, old, middle aged, very young, not young, not old, etc.*, which can be constructed from a principal value (for instance, *young*), its antonym (*old*) and negation (*not young*), and combining them by using semantic hedges like ‘very’. In sum, by shading the taken linguistic principal values. The concept of a Linguistic Variable was essential for Zadeh’s proposal of *Computing with Words and Perceptions (CwW)*, in the late 1990s; i.e., the establishment of a calculus allowing representation and reasoning, with precise and imprecise complex statements in either plain, or a semi-artificial, language, and based on fuzzy logic. It constitutes a step further along the path of Leibniz’s famous dictum, *Calculemus*!

The fourth and the fifth are ‘technical’, in the sense of allowing two important operative problems to be solved: those of obtaining logical consequences from a set of imprecise premises, and extending classical operations to the fuzzy field, respectively. The first is essential for the problem which underlies Fuzzy Control; i.e., how to obtain the output resulting from a set of imprecise linguistic rules describing the behavior of a dynamic system and an observed fact thereof. The second was fundamental for extending the arithmetical operations of addition, rest, multiplication and division, to fuzzy numbers for computing, for instance, ‘around four’ plus ‘around nine’, etc., and allowing the establishment of an Arithmetic of fuzzy numbers, as well as for handling linguistic expressions such as ‘the probability that John is young is high’. The Extension Principle is widely used in problems that concern CwW, along with the extensive use of Linguistic Variables and Generalized Constraints.

4. Conclusion

ET. In my own view, the most remarkable thing in Zadeh’s post-1965 work is that it meant something like the opening of a door towards scientific analysis and the technological exploitation of linguistic imprecision. This is work which allows representation through sets, attained thanks to the work George Boole embarked on the mid-19th Century, to be extended to other fields.

Language is permeated by such a large amount of imprecise words that the impossibility of their mathematical representation through the Boolean algebra of classical or crisp sets, based in its ‘specification’ axiom, is a powerful limitation (even if it is not the only one) on the possibility of first, mechanizing plain language, and
second, commonsense or ordinary reasoning. Without fuzzy sets and fuzzy relations, it is not actually possible to get ‘machines thinking like people’, in Zadeh’s own words, with precisely used terms; and this limitation is too strong for capturing many of the nuances of plain language. Ortholattices, De Morgan algebras, and even the weaker typical Standard algebras of fuzzy sets, are successful for some parts of language, but are endowed with laws and properties that cannot always be supposed in plain language and ordinary reasoning. In addition, it is not evident that Kolmogorov’s theory of probability is applicable to the not always random uncertainty appearing in linguistic statements, owing essentially to the impossibility in many cases of assuming the additive law, and even though ‘imprecise events’ may admit perfect classifications or partitions. Often, with imprecise events, a totality is not recognizable as a ‘sum’ of disjoint parts; for instance, Zadeh’s measures of possibility are not additive. The loss and gain of imprecise information cannot be seen as it can be with precise information.

Without the possibility of representing plain language and ordinary reasoning, it will be very difficult (if at all possible) to untie the famous ‘Gordian Knot’ of Artificial Intelligence. In my view, the true future for such a goal lies in continuing along the lines of ‘Computing with Words and Perceptions’ established by Zadeh. Nonetheless, beforehand, we will need to learn more about the natural phenomena of language, reasoning and creativity; and, to my way of understanding, what would help to do so would be to develop a type of ‘physics’ for these three natural phenomena [6].

Just as Physics deals with matter, energy, and movement, CwW should address an experimentally based theoretic science which, up until now, has not existed. In order to attain such a science, significant experiments have yet to be designed and, perhaps to begin with, these could be conducted through searching for the actual uses of linguistic connectives, hedges, quantifiers, relations of inference and analogy, etc., on the Internet [2]. This, at least, would serve to refine the use of the mathematical operations usually employed to design them; since the meaning of words in plain language is always context-dependent and purpose-driven, fuzzy sets are not only a matter of degree but, at least with regard to their application to reality, it is also a matter of design. Badly designing the membership functions representing fuzzy terms could easily lead to solving a problem other than the one currently posed.

AS. I don’t believe that there will ever be a single ‘calculus of language’, but who knows if there could be a family of calculus for each specific part of language. Ultimately, Boolean algebras are useful for representing precise reasoning, with either random or possibilistic types of uncertainty; orthomodular lattices serve for the uncertain case of Quantum Physics; Standard algebras of fuzzy sets take care of several cases of imprecise and uncertain reasoning, etc. A calculus of language would require a combination of logics, but at the present moment in time, that challenge would seem to be a chimera. In any case, a good deal of the history of science and technology lies in the pursuit of utopias.

ET. Fundamentally, Zadeh poses an intriguing problem to researchers, concerning not only the modeling and representation of plain language and ordinary reasoning [14, 13], but also its exploitation for arriving at machines that will actually think like people. That is, he confronts us with the ultimate problem of Artificial Intelligence.

AS. AI has to address that ascribed to Seymour Papert’s ‘superhuman human fallacy’ [1], which says that a machine does not think unless it shows superhuman skills. But human intelligence is characterized by providing, through the processes of ‘trial and error’, partial, revisable and provisional solutions to problems that are frequently unsharp. In this task, tolerant and robust systems will be desirable, and Zadeh’s fuzzy logic provides interesting tools for contribute to this aim.
References


